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The 's-rule' exclusion principle and vacuum interpolation in worldvolume dynamics

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Abstract: We show how the worldvolume realization of the Hanany-Witten effect for a supersymmetric D5-brane in a D3 background also provides a classical realization of the 's-rule' exclusion principle. Despite the supersymmetry, the force on the D5-brane vanishes only in the D5 'ground state', which is shown to interpolate between 6-dimensional Minkowski space and an OSp(4*|4)-invariant $AdS_2 \times S^4$ geometry. The M-theory analogue of these results is briefly discussed.

Keywords: Superstrings and Heterotic Strings, D-branes, Supersymmetry and Duality.

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1. Introduction

The worldvolume dynamics of a probe brane in the supergravity background of another brane provides a useful way to understand certain interactions between the two string/M theory branes. Consider the case of a D5-brane in the presence of N coincident D3-branes, aligned so as to preserve 1/4 supersymmetry according to the array

$$\begin{array}{lcccccccccc} N\,D3: & 1 & 2 & 3 & - & - & - & - & - & - \\ \text{probe } D5: & - & - & - & 4 & 5 & 6 & 7 & 8 & - \end{array} \tag{1.1}$$

For large N we may replace the D3-branes by the supergravity D3-brane since the curvature of this 1/2 supersymmetric solution of IIB supergravity is everywhere small in this limit, and the constant dilaton may be chosen such that the IIB string coupling is small. We should therefore expect the worldvolume dynamics of a probe D5-brane in this spacetime to capture effects associated to the interaction of the D5-brane with the N D3-branes.

One such effect was pointed out (in a dual context) by Hanany and Witten [1]: if the D5-brane is initially separated from the D3-branes along the 9-axis then a IIB string stretching between the D5-brane and each of the N D3-branes is created as the D5-brane is pulled through the D3-branes. The final, and still 1/4 supersymmetric, brane-plus-string configuration can be represented by the array

$$\begin{array}{lcccccccccc} N\,D3: & 1 & 2 & 3 & - & - & - & - & - & - \\ \text{probe } D5: & - & - & - & 4 & 5 & 6 & 7 & 8 & - \\ N \text{ strings}: & - & - & - & - & - & - & - & - & 9 \end{array} \tag{1.2}$$

The worldvolume description of this 'string creation' effect in terms of the worldvolume dynamics of the D5-brane was initiated by Callan et al. [2], who solved an equation found by Imamura for an S^5-wrapped D5-brane in $AdS_5 \times S^5$ [3], which is the near-horizon limit of the D3-brane geometry [4]. This provides a worldvolume realization of Witten's baryon vertex [5] but, for reasons that we will explain later, the Hanany-Witten (HW) effect can only be properly understood in the context of the *full* D3 geometry. An extension of Imamura's equation to the full D3-brane geometry was also proposed and analysed numerically in [2], but the status of this equation only became clear in subsequent works in which it was recovered from the conditions of minimal energy [6] and preservation of 1/4 supersymmetry [7], and solved analytically [8].

Note that the array (1.2) corresponds to a supersymmetry preserving configuration only for one orientation of the N strings; given the orientations of the D3-branes and the probe D5-brane (i.e., a choice of brane vs. anti-brane in each case) only one of the two possible string orientations (string or anti-string) is compatible with supersymmetry. As the D5-brane is passed through the D3-branes, starting from the configuration with N strings represented by (1.2), the orientation of the strings that connect them would be reversed, and hence supersymmetry would be broken, if it were not for the fact that these strings are destroyed by the 'reverse' Hanany-Witten effect, which returns us to the configuration without strings represented by the array (1.1). This escape from contradiction fails if any of the D5-branes is connected to a D3-brane by more than one string, and this led Hanany and Witten to propose (for their dual brane setups) that any such multi-string configuration would break supersymmetry [1]. This 's-rule' can be understood as a quantum effect in IIB superstring theory: the ground state of a string stretched between a D5-brane and a totally orthogonal D3-brane is fermionic, so the Pauli exclusion principle forces any additional strings into non-supersymmetric states of higher energy [9]. One aim of this paper is to show how the s-rule also has a *classical* explanation in terms of D5-brane worldvolume dynamics; this is similar in spirit (but quite different in detail) to the classical interpretation of the T-dual $D2\perp D6$ s-rule in terms of the worldvolume dynamics of M2-branes in M-theory [9] (see also [10]).

As just explained, the equations relevant to the $D5\perp D3$ setup under consideration here have been found and solved in previous studies of the HW effect. However, these previous analyses are incomplete in several respects. To explain why we must describe some features of the function $Z(\rho)$ that gives the position Z on the 9-axis of the D5 probe as a function of radial distance ρ on the probe. This function depends on two integration constants, a distance Z_∞ which gives the separation of the asymptotic planar D5-brane from the D3-branes, and a constant ν that is linearly related to the Born-Infeld (BI) electric charge as measured by the flux at infinity. Specifically, $Z(\rho)$ is given implicitly by the equation[1]

$$Z = Z_\infty + \frac{L^4}{2\rho^3}\left[\arctan\left(\frac{\rho}{Z}\right) - \frac{\rho Z}{\rho^2 + Z^2} - \pi\nu\right], \tag{1.3}$$

where L is the 'size' of the D3-brane core. For $0 \leq \nu \leq 1$, the function $Z(\rho)$ describes, for *positive* Z_∞, a D5-brane connected by νN strings to the D3-branes; the $\nu = 0$ case corresponds to the array (1.1) (with no strings) and the $\nu = 1$ case to the array (1.2) (with

[1]The arctan function here takes values in the interval $[0, \pi]$.

N strings). These two solutions are interchanged by taking $Z_\infty \to -Z_\infty$, so they actually belong to a single *family* of solutions (depending on Z_∞) that provides a worldvolume realization of the HW effect. Analogous families of solutions with $\nu > 1$ have not been considered previously. This neglect was possibly motivated by the s-rule, which might seem to suggest that solutions with $\nu > 1$ must be unphysical. However, we shall show here that the $\nu > 1$ solutions have a simple physical interpretation, again in terms of νN strings emerging from the D5-brane, but at most N of these strings end on the D3-brane, thus confirming the s-rule.

Although integer ν yields D5-brane geometries that have a simple interpretation in terms of attached strings, non-integer ν is also possible because the D5-brane is infinite. Taking into account the freedom represented by the integration constant Z_∞, one can restrict ν to the range

$$\nu \geq \frac{1}{2} \tag{1.4}$$

without loss of generality. The minimal $\nu = 1/2$ case is of particular interest but many of its special properties have been overlooked previously. Another aim of this paper is to provide a more complete treatment of this case. In particular, we show that the induced metric on the D5-brane interpolates between six-dimensional Minkowski space (as $\rho \to \infty$) and $AdS_2 \times S^4$ (as $\rho \to 0$). Moreover, the BI fields vanish in both limits, so we have a vacuum interpolation 'on the brane' analogous to the interpolation noted in [4] for the D3 background. In confirmation of this interpretation we show that the $AdS_2 \times S^4$ D5-brane vacuum has double the number of supersymmetries of the interpolating D5-brane. In fact, it is invariant under the transformations generated by the supergroup OSp($4^*|4$), which was identified in [6] as the 'ground-state' supergroup but without proper identification of the corresponding D5-brane configuration.

We should note at this point that the $AdS_2 \times S^4$ D5-vacuum has recently been discussed [11, 12] in the context of the AdS/dCFT correspondence [13, 14]; it can be viewed as the near-D3-horizon limit of the $Z_\infty = 0$ case of a $\nu = 1/2$ D5-brane; the OSp($4^*|4$) symmetry can be interpreted as conformal supersymmetry for an $\mathcal{N} = 8$ superconformal quantum mechanics. By taking $Z_\infty \neq 0$ we then find supersymmetric (but non-conformal) deformations of $AdS_2 \times S^4$ with potential implications for the AdS/dCFT correspondence.

In addition to providing a much simplified derivation of (1.3) from supersymmetry, we also present a simplified formula for the energy density, which we use to interpret the results of our analysis of (1.3), and a new formula for the net force exerted on the D5-brane by the D3-branes. Surprisingly, this force does not generally vanish despite the fact that the D5-brane configuration is both static and supersymmetric! This is possible because a finite force cannot move an object of infinite mass such as an infinite 5-brane. Whenever it would be possible to compactify the D5-brane (for example, by periodic identification) then the force must vanish because the total mass on which it acts would then be finite and the D5-brane otherwise could not be static. Such a compactification is possible only if $\nu = 1/2$ (because only in this case is there no BI flux at infinity). We find that the force vanishes precisely in this case, but not otherwise.

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We will begin with a re-derivation of the result (1.3) from supersymmetry that incorporates significant simplifications, mainly due to a better gauge choice. We then reconsider the energy of the D5-brane and compute the force on it. In the subsequent section we review the worldvolume interpretation of the HW effect for the $\nu = 1$ case and extend the analysis to $\nu > 1$; this yields our worldvolume interpretation of the s-rule. We then turn to the $\nu = 1/2$ case and demonstrate its vacuum interpolation property, and the enhanced supersymmetry of the $AdS_2 \times S^4$ embedding in $AdS_5 \times S^5$. We leave to a final section a discussion of how similar results apply to a probe M5-brane in an M5 background, and implications for AdS/dCFT.

2. Baryonic D5-brane revisited

We consider a 1/2 supersymmetric D3 background solution of IIB supergravity for which the dilaton is constant and the only non-zero fields are the metric and Ramond-Ramond (RR) 5-form field strength R_5. Choosing cylindrical polar coordinates for the transverse $\mathbb{E}^6$ space, we have the metric

$$ds^2_{10} = U^{-1/2}\left[-dT^2 + d\vec{X}\cdot d\vec{X}\right] + U^{1/2}\left[d\Upsilon^2 + \Upsilon^2 d\Omega_4^2(\Xi) + dZ^2\right], \tag{2.1}$$

where $\vec{X}$ are $\mathbb{E}^3$ cartesian coordinates, Υ is the radial coordinate in $\mathbb{E}^5$ and $d\Omega_4^2(\Xi)$ is the SO(5)-invariant metric on the unit 4-sphere, parametrized by four angles $\{\Xi\}$. The function U is given by

$$U = 1 + \frac{L^4}{(\Upsilon^2 + Z^2)^2}, \tag{2.2}$$

where the D3-brane core size L is given in terms of the integer N, the IIB string coupling constant g_s and the 'fundamental' IIB string tension T_f by

$$L^4 = \frac{g_s N}{\pi T_f^2}. \tag{2.3}$$

The RR 5-form is

$$R_5 = 4L^4\left[\omega_5 + \star\omega_5\right] \tag{2.4}$$

in polar coordinates for $\mathbb{E}^6$, where ω_5 is the volume 5-form on the unit 5-sphere and $\star\omega_5$ is its 10-dimensional Hodge dual. In our cylindrical polar coordinates,

$$\begin{aligned}\omega_5 &= \sin^4\Theta\; d\Theta\wedge\omega_4\\ &= \frac{3}{8}d\left[\Theta - \sin\Theta\cos\Theta - \frac{2}{3}\sin^3\Theta\cos\Theta\right]\wedge\omega_4\,,\end{aligned} \tag{2.5}$$

where ω_4 is the volume 4-form on the unit 4-sphere, and

$$\tan\Theta = \frac{\Upsilon}{Z}. \tag{2.6}$$

Given an asymptotically flat D5-brane in this D3 background, we may choose worldvolume coordinates $x^i = (t, \rho, \xi)$, where $\{\xi\}$ are four angular coordinates for the 4-sphere at fixed radial distance ρ from a worldspace origin. The worldvolume diffeomorphisms may now be fixed by the gauge choice

$$T = t\,, \qquad \Upsilon = \rho\,, \qquad \{\Xi\} = \{\xi\}\,. \tag{2.7}$$

This leaves $\vec{X}$ and Z as the worldvolume fields determining the geometry of the D5-brane. Given the static SO(5)-invariant ansatz

$$\vec{X} \equiv 0, \qquad Z = Z(\rho)\,, \tag{2.8}$$

the induced worldvolume metric g is

$$ds^2(g) = -U^{-1/2}(\rho)\; dt^2 + U^{1/2}(\rho)\left\{\left[1 + (Z')^2\right] d\rho^2 + \rho^2 d\Omega_4^2(\xi)\right\} \tag{2.9}$$

where the prime indicates differentiation with respect to ρ and, now,

$$U(\rho) = 1 + \frac{L^4}{\left[\rho^2 + Z^2(\rho)\right]^2}\,. \tag{2.10}$$

We must also take into account the worldvolume Born-Infeld field strength $F = dV$. Given the ansatz

$$V = \Phi(\rho) dt \tag{2.11}$$

we have a radial electric field $E = \Phi'$.

Let $\mathcal{R}_5$ be the pullback of R_5 to the worldvolume; the D5-brane worldvolume action in the chosen background is then

$$I = -T_5 \int d^6x\, \sqrt{-\det(g+F)} \;+\; T_5 \int \mathcal{R}_5 \wedge V\,, \tag{2.12}$$

where T_5 is the D5-brane tension, given in terms of the inverse string tension and the string coupling constant g_s by

$$T_5 = \frac{T_f^3}{4\pi^2 g_s}\,. \tag{2.13}$$

For our gauge choice and ansatz we have

$$\sqrt{-\det(g+F)} = \rho^4 U \sqrt{1 + (Z')^2 - E^2}\; \mathrm{vol}_4(\xi)\,. \tag{2.14}$$

where $\mathrm{vol}_4(\xi)$ is the volume scalar density on the unit 4-sphere. To similarly simplify the remaining (Wess-Zumino) term in the action, we first observe that the pullback of $\star\omega_5$ vanishes because $d\vec{X} \equiv 0$; then, using (2.5), we find that

$$\mathcal{R}_5 \wedge V = -4L^4 \Phi\; dt \wedge d\left[\theta - \sin\theta\cos\theta - \frac{2}{3}\sin^3\theta\cos\theta\right] \wedge \omega_4\,, \tag{2.15}$$

where the function $\theta(\rho)$ is determined in terms of $Z(\rho)$ through the relation[2]

$$\tan\theta = \frac{\rho}{Z}\,. \tag{2.16}$$

[2]A similar function was introduced in [15] for a D5-brane in $AdS_5 \times S^5$.

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The integral over the angular variables $\{\xi\}$ in (2.12) is now trivially done and yields a factor of $8\pi^2/3$ (this being the volume of the unit 4-sphere); discarding a total derivative, we are then left with the effective Lagrangian density

$$\mathcal{L} = -\frac{8\pi^2 T_5}{3}\left\{\rho^4 U\sqrt{1+(Z')^2-E^2} - \frac{3}{2}L^4 E\left[\theta - \sin\theta\cos\theta - \frac{2}{3}\sin^3\theta\cos\theta\right]\right\}. \qquad (2.17)$$

The equation of motion for the BI field Φ yields the Gauss law constraint for E, which can be integrated immediately to give

$$\frac{U\rho^4 E}{\sqrt{1+(Z')^2-E^2}} = \frac{3}{2}L^4\left[\pi\nu - \theta + \sin\theta\cos\theta + \frac{2}{3}\sin^3\theta\cos\theta\right], \qquad (2.18)$$

where ν is an integration constant.

We now proceed to determine the conditions required by partial preservation of supersymmetry. Let $\Gamma_A = (\Gamma_T, \Gamma_{\vec{X}}, \Gamma_\Upsilon, \Gamma_Z, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4)$ be constant Dirac matrices, with the last four associated in the standard way to the four angles $\{\Xi\}$ parametrizing S^4. Let χ be a covariantly constant $\mathrm{Sl}(2;\mathbb{R})$-doublet chiral spinor in the D3 background. Such spinors take the form

$$\chi = U^{-1/8}\epsilon\,, \qquad (2.19)$$

where ϵ is a *Minkowski* space covariantly constant spinor subject to the '$D3$ constraint'

$$i\sigma_2 \otimes \Gamma_T\Gamma_{X_1}\Gamma_{X_2}\Gamma_{X_3}\ \epsilon = \epsilon\,, \qquad (2.20)$$

where σ_2 is the 2×2 Pauli matrix. Owing to the chiral nature of ϵ, this is equivalent to

$$i\sigma_2 \otimes \Gamma_Z\Gamma_\Upsilon\Gamma_*\ \epsilon = \epsilon\,, \qquad (2.21)$$

where

$$\Gamma_* = \Gamma_1\Gamma_2\Gamma_3\Gamma_4\,. \qquad (2.22)$$

Note that Γ_* commutes with Γ_T, Γ_Υ and Γ_Z, and is such that $\Gamma_*^2 = 1$.

In the presence of a D5-brane there is an additional constraint on ϵ of the form $\Gamma_\kappa\epsilon = \epsilon$, where Γ_κ is the kappa-symmetry matrix of the super D5-brane. In the conventions of [16], and for a purely electric BI field, this additional condition is

$$\sqrt{-\det(g+F)}\ \epsilon = \left[\sigma_1\otimes\Sigma - iE\sigma_2\otimes\gamma^t\gamma^\rho\Sigma\right]\epsilon\,, \qquad (2.23)$$

where γ_i are the induced worldvolume Dirac matrices, and

$$\Sigma = \frac{1}{6!}\varepsilon^{ijklmn}\gamma_i\gamma_j\gamma_k\gamma_l\gamma_m\gamma_n\,. \qquad (2.24)$$

Given spacetime frame 1-forms $E^A = dX^M E_M{}^A$, we have $\gamma_i = \partial_i X^M E_M{}^A\Gamma_A$. For the obvious choice of zehn-bein $E_M{}^A$ we find that

$$\Sigma = U\rho^4\Gamma_T\Gamma_*\left[\Gamma_\Upsilon + Z'\Gamma_Z\right]\ \mathrm{vol}_4(\xi)\,, \qquad (2.25)$$

and

$$\gamma^t \gamma^\rho = -\left[1 + (Z')^2\right]^{-1} \Gamma_T \left[\Gamma_\Upsilon + Z'\Gamma_Z\right]. \tag{2.26}$$

Given also (2.14), we then deduce, after some algebra, that[3]

$$\sqrt{1 + (Z')^2 - E^2}\;\epsilon = \sigma_1 \Gamma_T \Gamma_* \Gamma_\Upsilon\;\epsilon + i\sigma_2 \Gamma_* \left(E - Z'\sigma_3 \Gamma_T \Gamma_Z\right)\epsilon\,. \tag{2.27}$$

Note that the function U has cancelled, so the final result must be the same as for flat space!

The constraint (2.27) must be satisfied for all ρ. Because E and Z' vanish asymptotically, as $\rho \to \infty$, we deduce from (2.27) the 'D5-constraint'

$$\sigma_1 \Gamma_T \Gamma_* \Gamma_\Upsilon\;\epsilon = \epsilon\,. \tag{2.28}$$

This is compatible with the D3-constraint (2.21) and reduces the fraction of supersymmetry preserved to 1/4. Given that ϵ also satisfies (2.21), the supersymmetry preserving condition can now be reduced to

$$\left[\sqrt{1 + (Z')^2 - E^2} - 1\right]\epsilon = (E - Z')\Gamma_\Upsilon \Gamma_Z \epsilon\,. \tag{2.29}$$

In the cylindrical polar coordinates used here, $\epsilon = M(\xi)\epsilon_0$ for constant spinor ϵ_0 and matrix function M of the 4-sphere angles $\{\xi\}$. As M does not commute with Γ_Υ, the equation (2.29) can be satisfied for all $\{\xi\}$ if and only if

$$E = Z'\,. \tag{2.30}$$

Thus, any static SO(5)-invariant D5-brane configuration with $E = Z'$ preserves 1/4 supersymmetry.

Using $E = Z'$ in the integrated Gauss law constraint (2.18), we deduce that

$$U\rho^4 Z' = -\frac{3}{2} L^4 \left[\theta - \sin\theta\cos\theta - \frac{2}{3}\sin^3\theta\cos\theta - \pi\nu\right]. \tag{2.31}$$

Given the form (2.10) of the function U, and the relation (2.16) between the functions Z and θ, it can be shown [8] that this is equivalent to

$$Z' = \left[\frac{L^4\left(\theta - \sin\theta\cos\theta - \pi\nu\right)}{2\rho^3}\right]'. \tag{2.32}$$

This is trivially integrated, and the result is the implicit equation (1.3) for Z quoted in the introduction, which we may write as

$$Z = Z_\infty + \frac{L^4 \eta_\nu(\theta)}{2\rho^3}\,, \tag{2.33}$$

where

$$\eta_\nu(\theta) = \theta - \sin\theta\cos\theta - \pi\nu\,. \tag{2.34}$$

[3]Here, and henceforth, we supress the tensor product symbol.

3. Energy and Force

The interpretation of our results to follow will rely on a formula for the D5-brane energy that we now derive. The effective D5-brane Hamiltonian density is

$$\mathcal{H} \equiv E\frac{\partial\mathcal{L}}{\partial E} - \mathcal{L} = \frac{8\pi^2}{3}T_5\,\frac{\rho^4 U\left[1+(Z')^2\right]}{\sqrt{1+(Z')^2-E^2}}\,. \tag{3.1}$$

Setting $E = Z'$ and using (2.31), we find that

$$\mathcal{H} = \left(\frac{8\pi^2 T_5}{3}\right)\rho^4 U + Z' D_\nu(\theta) \tag{3.2}$$

for supersymmetric D5-branes, where

$$D_\nu = \frac{NT_f}{\pi}\left[\pi\nu - \theta + \sin\theta\cos\theta + \frac{2}{3}\sin^3\theta\cos\theta\right]. \tag{3.3}$$

Here we have used the relation

$$4\pi^3 T_5 L^4 = NT_f\,, \tag{3.4}$$

which follows from (2.3) and (2.13). The first term in (3.2) is the energy density due to the D5 surface tension. The second term is due to the BI electric field, and its form shows that D_ν can be interpreted as a tension along the Z-axis, at least whenever it is approximately constant. This observation is crucial to the interpretation of the 1/4 supersymmetric D5-brane configurations, on which we elaborate in the following sections.

The formula for the energy density (3.2) can also be written in the form [6]

$$\mathcal{H} = \mathcal{H}_0 + \left[\left(\frac{2NT_f}{3\pi}\right)\rho\sin^4\theta + ZD_\nu\right]'\,, \tag{3.5}$$

where

$$\mathcal{H}_0 = \frac{8\pi^2}{3}T_5\rho^4 \tag{3.6}$$

is the energy density for a flat vacuum D5-brane in flat space. The total energy

$$H = \int_0^\infty d\rho\,\mathcal{H} \tag{3.7}$$

is of course infinite. One can subtract the infinite integral of $\mathcal{H}_0$, but the remainder is still infinite because of the term linear in ρ. One can get a finite result by considering the derivative with respect to Z_∞. Noting that

$$\frac{dZ}{dZ_\infty} = U^{-1}\,, \tag{3.8}$$

one can show that

$$\frac{d\mathcal{H}}{dZ_\infty} = \frac{2NT_f}{3\pi L^4}\left[U^{-1}D_\nu\right]'\,. \tag{3.9}$$

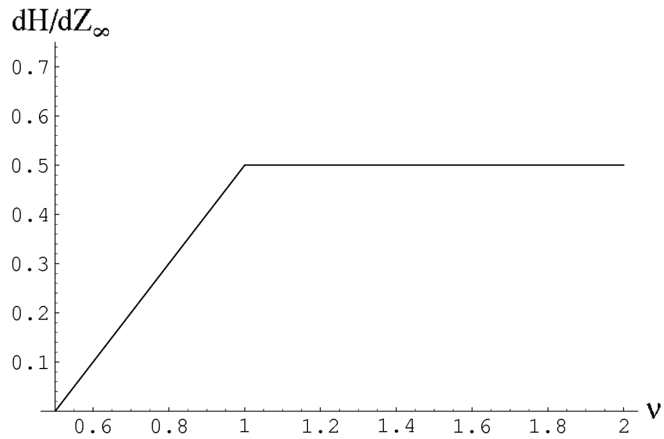

Figure 1: The net force (in units NT_f) on the D5-brane as a function of ν.

Using (2.31), and integrating over ρ, we deduce that

$$\frac{dH}{dZ_\infty} = \frac{2NT_f}{3\pi L^4}\left[\rho^4 Z'\right]_0^\infty . \tag{3.10}$$

One sees immediately from (2.33) that

$$\rho^4 Z' \to \frac{3\pi L^4}{2}\left(\nu - \frac{1}{2}\right) \tag{3.11}$$

as $\rho \to \infty$. An analysis of the behaviour of Z as $\rho \to 0$ will be considered in more detail in the following sections. We will see that, as $\rho \to 0$,

$$\rho^4 Z' \to \begin{cases} 0 & \text{for } \frac{1}{2} \le \nu \le 1 \\ \frac{3\pi L^4}{2}(\nu - 1) & \text{for } \nu > 1 \end{cases} . \tag{3.12}$$

This yields the result

$$\frac{dH}{dZ_\infty} = \begin{cases} NT_f\left(\nu - \frac{1}{2}\right) & \text{for } \frac{1}{2} \le \nu < 1 \\ \frac{1}{2}NT_f & \text{for } \nu \ge 1 \end{cases} . \tag{3.13}$$

Thus, the net force vanishes *only* for $\nu = 1/2$. Figure 1 shows a plot of the force dH/dZ_∞ as a function of ν.

As $E = Z'$, the asymptotic behaviour of Z' given in (3.11) shows that the BI electric charge as measured by the electric flux at infinity is proportional to $(\nu - 1/2)$. This vanishes for $\nu = 1/2$, so in this case there is no obstruction to a compactification of the D5-brane. We could compactify on a torus and then T-dualize to the D0-D8 system, for which the force is known to vanish [17], so we should find a vanishing force on the D5-brane when $\nu = 1/2$, and we do. When $\nu \neq 1/2$ no such argument applies and, as a compactification is not possible, the total mass of the D5-brane is necessarily infinite. As an infinite mass cannot be moved by a finite force, a non-zero force is compatible with the fact that the D5-brane is static. As we have seen, it is also compatible with supersymmetry.

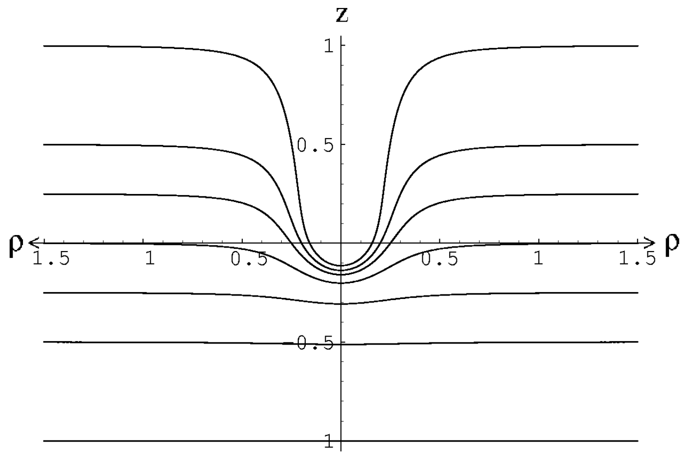

Figure 2: $\nu = 1$: $Z(\rho)$ (and its mirror image) for various values of $Z_\infty \in [-1, 1]$.

4. String creation and the s-rule

The key equation (2.33) is equivalent to

$$\tilde{Z} = \tilde{Z}_\infty + \frac{L^4 \eta_{(1-\nu)}(\tilde{\theta})}{2\rho^3}, \tag{4.1}$$

where

$$\tilde{Z} = -Z, \qquad \tilde{Z}_\infty = -Z_\infty, \qquad \tilde{\theta} = \arctan\left(\frac{\rho}{\tilde{Z}}\right). \tag{4.2}$$

This shows that there is no loss of generality in restricting ν in (2.33) to the range $\nu \geq 1/2$, as claimed in the introduction, as long as we allow Z_∞ to be either positive or negative. When $Z_\infty > 0$ and $\nu \leq 1$, the constant νN has the interpretation as the number of strings connecting the D5-brane to the N D3-branes (although, strictly speaking, this interpretation makes sense only for $\nu = 1$). But when $Z_\infty < 0$ one finds that the D5-brane is connected to the D3-branes by $(\nu - 1)N$ anti-strings, and this is responsible for the HW effect. The interpretation is simplest for $\nu = 1$, for which, following [8], we present plots of $Z(\rho)$ for various values of Z_∞, see figure 2. These plots clearly exhibit the mechanism underlying the effect; essentially, the $\rho < L$ region of the D5-brane remains trapped on a 5-sphere surrounding the D3-branes as the D5-brane is pulled through them, but this wrapped D5-brane remains connected to the asymptotic D5-brane by a tube of S^4 cross section that can be interpreted as N strings. This explains the HW effect for $\nu = 1$; a different explanation is needed for $\nu > 1$ and this will be provided below. When $Z \ll Z_\infty$ we can approximate (2.33) by

$$\eta_\nu(\theta) = -\frac{2Z_\infty}{L^4}\rho^3. \tag{4.3}$$

Defining a new independent variable r (radial distance in the $\mathbb{E}^6$ transverse to the D3-branes) by

$$\rho = r \sin\theta \tag{4.4}$$

we can rewrite the above equation as

$$r^3 = -\frac{L^4}{2Z_\infty}\left(\frac{\eta_\nu(\theta)}{\sin^3\theta}\right), \tag{4.5}$$

which is equivalent to the result of [2] for a D5-brane in $AdS_5 \times S^5$. For $\nu = 1$ there is a minimum value r_{min} of r, with $r = r_{min}$ for $\theta = \pi$, and

$$Z_\infty r_{\rm min}^3 \sim L^4 \,. \tag{4.6}$$

Clearly, the near-horizon approximation to the D3 geometry is valid only if $r_{\rm min} \ll L$, but this condition is satisfied only if

$$Z_\infty \gg L \,. \tag{4.7}$$

In other words, the 'near horizon' D5 probe geometry of (4.5), which describes a D5-brane wrapped on an S^5 in $AdS_5 \times S^5$ with N strings attached [2], can be interpreted as a D5-brane surrounding N D3-branes at a distance $r_{\rm min}$ only if the N strings connect to a *distant* planar D5-brane. However, string creation occurs as Z_∞ passes through zero, at which point the condition (4.7) must fail. We conclude that the near-horizon result (4.5) is actually *not* relevant to a worldvolume description of the HW effect, although it is still a useful tool in the analysis of (2.33).

We have been viewing Z and θ as functions of ρ that are related by (2.16). However one can view (2.16) as a single relation between three variables (Z, θ, ρ), any one of which may be chosen as the independent variable. For some purposes it is convenient to choose θ as the independent variable, in which case (2.16) can be interpreted as defining the function

$$\rho(\theta) = Z(\theta)\tan\theta \,. \tag{4.8}$$

Using this in (2.33) we deduce that the function $Z(\theta)$ is given implicitly by the relation

$$Z = Z_\infty + \frac{L^4\eta_\nu(\theta)\cot^3\theta}{2Z^3} \,. \tag{4.9}$$

The function $Z(\theta)$ is not necessarily single-valued but there is always a branch near $\theta = \pi/2$ for which $Z \approx Z_\infty$. The induced metric on this branch approaches the flat Minkowski metric as $\theta \to \pi/2$. Plots of $Z(\theta)$ are shown in figure 3 for $\nu = 1$ and $Z_\infty > 0$ or $Z_\infty < 0$, respectively. For $Z_\infty < 0$, one sees, as expected, that Z remains everywhere close to Z_∞. For $Z_\infty > 0$, however, $Z(\theta)$ is doubled-valued at $\theta = \pi/2$, and vanishes on the second branch. On this branch, Z has a minimum at $\theta = \pi$, where its value is small and negative. The two branches are connected by a region in which θ approaches a small minimum value $\theta_{\rm min} \sim (L/Z_\infty)^{4/3}$ at $Z(\theta_{\rm min}) = 3Z_\infty/4$. In this region the D5 geometry is that of a thin tube along the Z-axis with a variable-radius 4-sphere as its cross section, as shown in figure 4. There is a similar tube for all $\nu \geq 1/2$ (given $Z_\infty \gg L$). From (3.2) we see that the energy per unit length of this tube is proportional to D_ν, and from (3.3) we have, near $\theta = 0$,

$$D_\nu(\theta) = NT_f\nu + \mathcal{O}(\theta^5) \,. \tag{4.10}$$

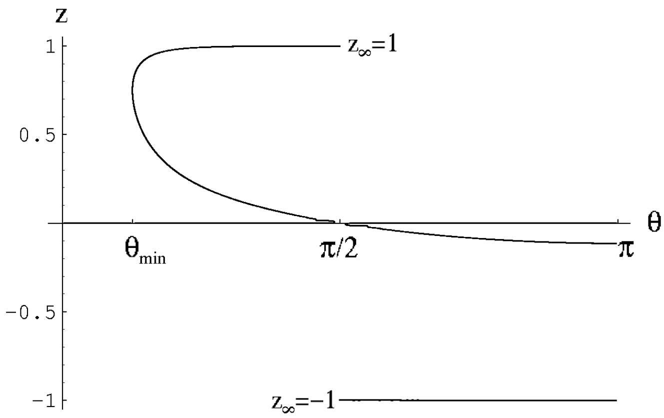

Figure 3: $\nu = 1$: $Z(\theta)$ for $Z_\infty = -1$ and $Z_\infty = 1$.

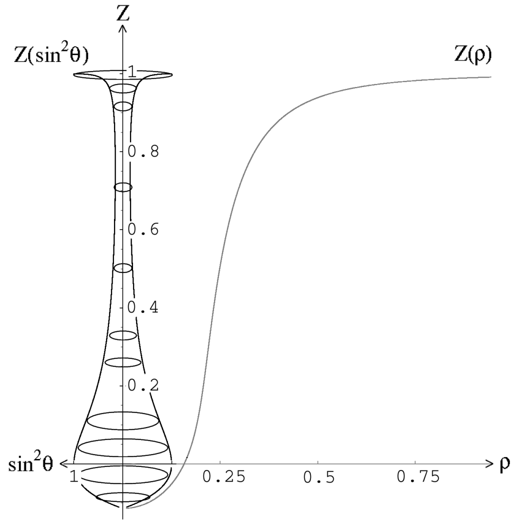

Figure 4: $\nu = 1$: $Z(\rho)$ and the string interpretation. Z as a function of the S^4-size $\sin^2\theta$.

The tube's tension is therefore $N\nu$ times the IIB string tension. For $\nu = 1$ we can therefore interpret the tube as N IIB strings stretched between the D5-brane and the N D3-branes. All previous analyses of the D5-brane worldvolume dynamics have been subject to the restriction $0 \leq \nu \leq 1$. Given that we may choose $\nu \geq 1/2$ without loss of generality, this means in effect that only the cases with

$$\frac{1}{2} \leq \nu \leq 1 \tag{4.11}$$

have been considered previously. The $\nu = 1$ case allows the simplest realization of the HW effect, as just reviewed. We have nothing new to say about the $\nu < 1$ cases, except for the special case of $\nu = 1/2$ which will be dealt with in the following section. This leaves the

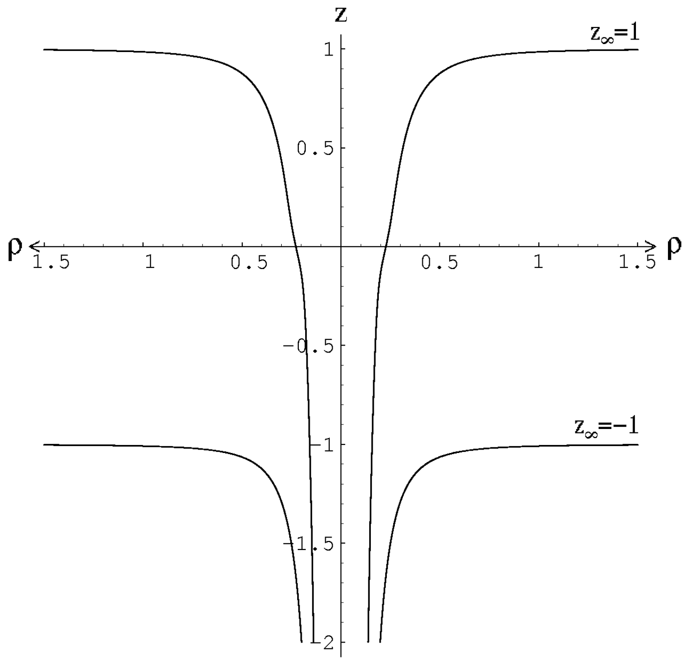

Figure 5: $\nu = 2$: $Z(\rho)$ (and its mirror image) for $Z_\infty = -1$ and $Z_\infty = 1$.

cases for which

$$\nu > 1\,. \tag{4.12}$$

For simplicity of presentation we shall assume that ν is an integer; this means that (when $Z_\infty > 0$) we have a D5-brane in the D3 background with νN strings attached to it. Our initial ansatz imposed an SO(5) symmetry which forces all these attached strings to lie along the axis ($\theta = 0, \pi$) separating the D5-brane from the D3-branes. Although one should take N large to justify the supergravity approximation, the results make formal sense for any N and it will be convenient to discuss the $N = 1$ case. We shall also assume, at least initially, that $Z_\infty > 0$. For $\nu = 1$ we then have a D5-brane geometry that can be interpreted as a D5-brane connected to a D3-brane by a single string. When $\nu > 1$ we must have ν strings that leave the D5-brane in the direction of the D3-brane (because any leaving in the opposite direction would be supersymmetry-breaking anti-strings). However, according to the s-rule, only one of these ν strings can end on the D3-brane, so *the other* ($\nu - 1$) *strings must pass through the D3-brane, without ending on it.* This conclusion may be verified qualitatively by inspection of the plot of the function $Z(\rho)$ for $\nu = 2$ (with $Z_\infty > 0$), see figure 5; comparison with the plot for $\nu = 1$ (figure 2) shows that at least one string now passes *through* the D3-brane. It might appear that all pass through the D3-brane but a closer analysis shows that this interpretation is not correct. Consider the plot of the function $Z(\theta)$ shown in figure 6 for $\nu = 2$. Here we see that in addition to the asymptotic region as $\theta \to \pi/2$ there is now *another asymptotic region* as $\theta \to \pi$. The geometry is that of an infinite BIon 'spike' [18, 19] along the Z-axis with cross section S^4. The 4-sphere has an ever-decreasing radius, but the energy per unit length is again

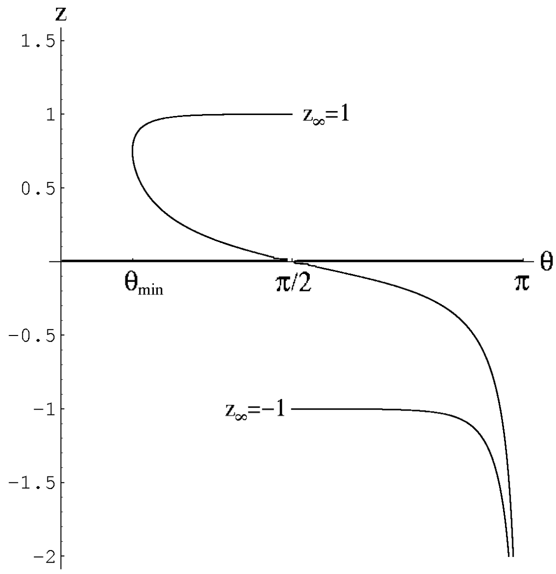

Figure 6: $\nu = 2$: $Z(\theta)$ for $Z_\infty = -1$ and $Z_\infty = 1$.

proportional to D_ν. As

$$D_\nu(\pi) = NT_f(\nu - 1)\,, \tag{4.13}$$

we may interpret this spike as $\nu - 1$ strings which have passed through the D3-brane without ending on it; see figure 7.

As we started with ν strings we must conclude, *despite the fact that the D5-brane does not cross the D3 horizon*, that one string has ended on the D3-brane.

The above discussion was for positive Z_∞ and $\nu > 1$. Turn now to the plot of the function $Z(\rho)$ for negative Z_∞ in figure 5. For $Z_\infty \gg L$ the D5-brane is always far from the D3-branes and the D5 probe geometry so there are now no strings connecting the D5-brane to the D3-brane. From the plot of $Z(\theta)$ in figure 6 one sees that there is again a second asymptotic region as $\theta \to \pi$, but this corresponds to $(\nu - 1)$ strings that leave the D5-brane in the *opposite direction to the D3-branes.* As Z_∞ is taken from positive to negative values the one string stretched between the D5 and the D3 is therefore destroyed, thus realizing the HW effect for $\nu > 1$. In no case is the D5 connected to the D3 by more than one string, thus confirming the s-rule.

5. Vacuum polarization and vacuum interpolation

For $1/2 \le \nu < 1$ and $Z_\infty \gg L$ there is always a 'near-horizon' branch of the function $Z(\theta)$ for which we may use the near-horizon relation (4.5). This shows that there exists a maximum value $\theta^{(\nu)}_{\rm max}$ of θ (with $\pi/2 \le \theta < \pi$, as illustrated in figure 8) and that

$$\theta \to \theta^{(\nu)}_{\rm max} \Rightarrow r \to 0\,, \tag{5.1}$$

where $r^2 = \rho^2 + Z^2$. This means that the D5-brane crosses the D3 horizon when ν lies in the range $1/2 \le \nu < 1$, in contrast to its behaviour for $\nu \ge 1$. We shall concentrate

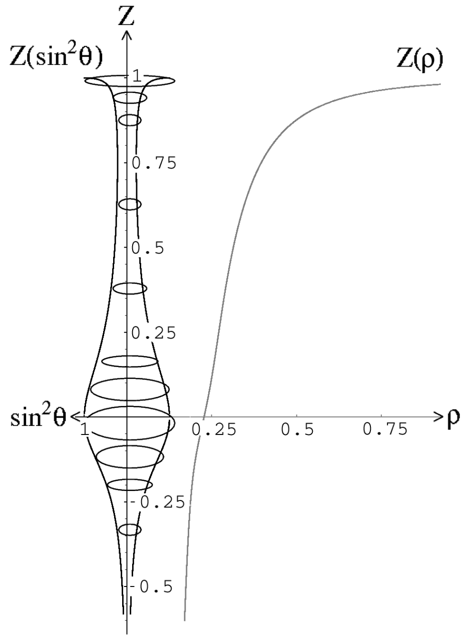

Figure 7: $\nu = 2$: $Z(\rho)$ and the string interpretation. Z as a function of the S^4-size $\sin^2\theta$.

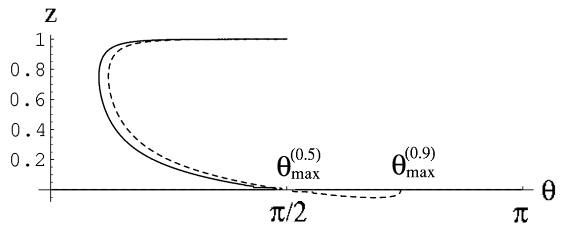

Figure 8: $\nu = 0.5$ and $\nu = 0.9$ (dashed): $Z(\theta)$ for $Z_\infty = 1$.

on the $\nu = 1/2$ case, for which $\theta_{\rm max}^{(0.5)} = \pi/2$. The function $Z(\theta)$ for this case is shown in figure 8 for $Z_\infty = 1$; note that there are *two* branches of the function at $\theta = \pi/2$, the near-horizon branch just discussed and an asymptotic branch on which the induced geometry is Minkowski.

From the formula (3.3) we see that

$$D_\nu(\pi/2) = N\left(\nu - \frac{1}{2}\right)T_f \tag{5.2}$$

and hence that $D_\nu \to 0$ asymptotically only if $\nu = 1/2$. In addition, when $\nu = 1/2$ we have $D_\nu \to 0$ on the near-horizon branch too! This means that $D_\nu(\theta)$ is non-vanishing only in the tubular region connecting the near-horizon and asymptotic branches of the D5-brane; in other words, *the D3-brane polarizes the electrically neutral D5-brane*, separating equal but opposite amounts of BI electric charge across the tubular region. The resulting

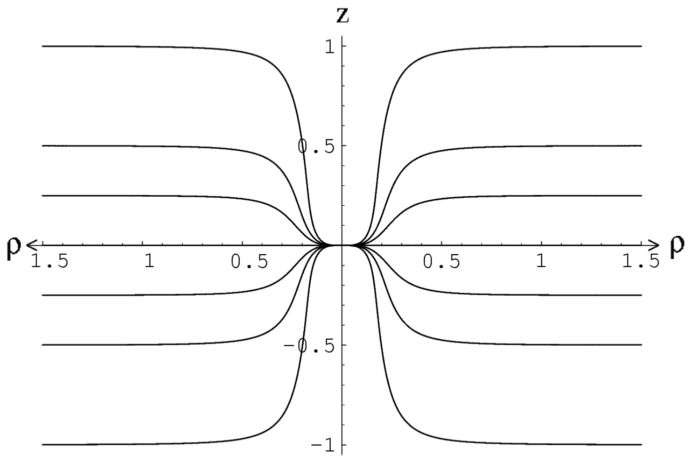

Figure 9: $\nu = 0.5$: $Z(\rho)$ (and its mirror image) for various values of $Z_\infty \in [-1, 1]$.

BI electric field can be interpreted as N half-strings. This observation provides a simple explanation of the HW effect for the $\nu = 1/2$ case. As Z_∞ is reduced from its large positive value, the polarized region shrinks until, at $Z_\infty = 0$, it vanishes (as we confirm below). As Z_∞ continues to decrease to large negative values the polarization reappears but now with the opposite orientation, thus creating N anti-half-strings. The net effect is to destroy N strings (or create them, depending on the initial choice of space orientation).

We shall now examine the D5-brane geometry for $\nu = 1/2$ in more detail. Setting $\nu = 1/2$ in (2.33) we have

$$Z = Z_\infty + \frac{L^4}{2\rho^3}\left[\theta - \frac{\pi}{2} - \sin\theta\cos\theta\right]. \tag{5.3}$$

This equation determines a family of functions $Z(\rho)$, or equivalently $\theta(\rho)$ with $\tan\theta = \rho/Z$, depending on the parameter Z_∞. As long as $Z_\infty \neq 0$, there is a branch of the function with $Z \approx 0$ that is described in the limit $\rho \to 0$ by the near-horizon approximation, which yields

$$\theta \sim \frac{\pi}{2} - \frac{Z_\infty}{L^4}\rho^3. \tag{5.4}$$

It follows from this that $\rho^4 Z' \to 0$ as $\rho \to 0$, as claimed in section 3.

It also follows that $Z \to 0$ as $\rho \to 0$. As can be seen from figure 9 the region in which $Z \approx 0$ grows as $Z_\infty \to 0$. This suggests that $Z \equiv 0$ when $Z_\infty = 0$. To verify this we must return to (5.3) and set $Z_\infty = 0$. The resulting equation indeed has the solution

$$Z_\infty \equiv 0. \tag{5.5}$$

To see that this is the *only* solution we note that if $Z_\infty = 0$ but $Z \neq 0$ then (5.3) implies that

$$Z^4 = -\frac{1}{2}L^4\cot^3\theta\left[\left(\frac{\pi}{2}\right) - \theta + \sin\theta\cos\theta\right] \leq 0, \tag{5.6}$$

which is impossible.

Let us now return to the generic $\nu = 1/2$ case, for which $Z \not\equiv 0$. Because $D_\nu(\pi/2) = 0$, we have $D_\nu \to 0$ as $\rho \to 0$. Using the near-horizon approximation for U we compute the induced metric in this limit to be

$$ds^2 = -\frac{\rho^2}{L^2}dt^2 + L^2\frac{d\rho^2}{\rho^2} + L^2 d\Omega_4^2\,. \tag{5.7}$$

This is an $AdS_2 \times S^4$ embedded in the near-horizon $AdS_5 \times S^5$ background with $E = Z' = 0$. We thus have a *worldvolume* analogue of the interpolation property of the D3-brane background [4]. Recall that in this case both the asymptotic Minkowski vacuum and the near-horizon $AdS_5 \times S^5$ vacuum are maximally supersymmetric, with twice the number of supersymmetries of the full D3 supergravity solution. We shall now show that the interpolating D5-brane has the same property by demonstrating that its $AdS_2 \times S^4$ vacuum has enhanced supersymmetry.

Our result of section 2 for the supersymmetry preserving constraint arising from the presence of the D5-brane is equivalent to the condition

$$\sigma_1\Gamma_T\Gamma_*\Gamma_\Upsilon\, \chi = \chi \tag{5.8}$$

on background Killing spinors χ. For the full D3 background these Killing spinors all take the form (2.19) but for the $AdS_5 \times S^5$ background there are additional Killing spinors of the form [20]

$$\chi = \left(\Upsilon^2 + Z^2\right)^{-3/4}\left[\Upsilon\Gamma_\Upsilon + Z\Gamma_Z - (\Upsilon^2 + Z^2)\left(T\Gamma_T + \vec{X}\cdot\vec{\Gamma}_X\right)\right]\eta\,, \tag{5.9}$$

where η is a covariantly constant spinor in the IIB Minkowski vacuum subject to the $\overline{D3}$ constraint

$$i\sigma_2\Gamma_Z\Gamma_\Upsilon\Gamma_*\,\eta = -\eta\,. \tag{5.10}$$

For our gauge choice and $\vec{X} \equiv 0$ ansatz we have

$$\chi = \left[r^{-3/2}\rho\Gamma_\Upsilon - r^{1/2}t\Gamma_T + r^{-3/2}Z\Gamma_Z\right]\eta\,. \tag{5.11}$$

Unless Z vanishes identically, the constraint (5.8) implies that $\eta = 0$. However, when $Z \equiv 0$ we find instead that η must satisfy

$$\sigma_1\Gamma_T\Gamma_*\Gamma_\Upsilon\,\eta = -\eta\,. \tag{5.12}$$

This is compatible with (5.10) and together these constraints imply that η has eight independent components. The spinor ϵ also has eight independent components so the number of supersymmetries is doubled when $Z \equiv 0$.

The isometry group of the embedding of $AdS_2 \times S^4$ in $AdS_5 \times S^5$ is a cover of $\mathrm{Sl}(2;\mathbb{R}) \times Sp_2$, which should therefore be a subgroup of the isometry supergroup, with the 16 charges in two $(\mathbf{2},\mathbf{4})$ representations of $\mathrm{Sl}(2;\mathbb{R}) \times Sp_2$. This supergroup must also be a subgroup of the $\mathrm{SU}(2,2|4)$ isometry supergroup of the background. The only candidate [6] is the supergroup

$$\mathrm{OSp}(4^*|4) \supset Spin^*(4) \times USp(4) \cong \mathrm{Sl}(2;\mathbb{R}) \times \mathrm{SU}(2) \times Sp_2 \tag{5.13}$$

for which the 16 supercharges are in the $(\mathbf{2},\mathbf{2},\mathbf{4})$ irreducible representation of $\mathrm{Sl}(2;\mathbb{R}) \times \mathrm{SU}(2) \times Sp_2$. The additional $\mathrm{SU}(2)$ factor has a natural interpretation as the rotation group acting on $\vec{X}$.

6. Discussion

The end result of a series of previous papers devoted to the D5 worldvolume interpretation of the Hanany-Witten effect is the implicit formula (1.3) that determines the geometry of a D5-brane in the D3 background. Here we have given a much simplified derivation of this formula from the condition for preservation of supersymmetry. The simplification is largely due to a better gauge choice, one that is adapted to the full D3 geometry rather than its near-horizon limit. Indeed, one of the lessons of our work that was not fully appreciated previously is that the full D3 geometry is needed for a worldvolume interpretation of the HW effect. Our new perspective on this problem has the virtue that it allows a straightforward physical interpretation of the *general* supersymmetric SO(5)-invariant 'baryonic' D5-brane in a D3 background, with arbitrary numbers of attached IIB strings (corresponding to arbitrary ν); in agreement with the 's-rule', we have found that at most one of these strings may end on the D3-brane. This provides a classical interpretation, in the spirit of [9], for what is usually interpreted as a quantum effect in IIB string theory due to the Pauli exclusion principle.

In the special case corresponding to $\nu = 1/2$, for which the D5-brane has no net BI electric charge, the D5-brane must cross the D3 horizon. The near horizon solution was found already in [2] but its AdS geometry was not previously apreciated, nor the fact that this solution has enhanced supersymmetry. Its OSp($4^*|4$) isometry supergroup was discussed previously [6], but without reference to the solution that actually exhibits it. Here we have presented what we hope is a complete account of this special D5-brane embedding in $AdS_5 \times S^5$. In fact, this special case was the starting point of our work; it is not difficult to see that an $AdS_2 \times S^4$ embedding must exist, and it was recently argued that it should have an interpretation, via the AdS/dCFT correspondence [13, 14], as a point defect in $\mathcal{N} = 4$ SYM-theory [11, 12]. If so, one might expect this defect to support an $\mathcal{N} = 8$ supersymmetric conformal quantum mechanics (SCQM), and one might expect supersymmetric deformations of the $AdS_2 \times S^4$ D5-brane to correspond to non-conformal perturbations of this SCQM. However, the supersymmetric deformations are the D5-brane geometries with $Z_\infty \neq 0$. These asymptote to $AdS_2 \times S^4$ as $\rho \to 0$, which corresponds to the *IR limit* of the putative SCQM; there seems to be no supersymmetric deformation that is asymptotic to $AdS_2 \times S^4$ in what would be the UV limit.

Our improved analysis of the D5-brane energy allowed us to compute the force on the D5-brane as a function of the parameter ν. This force need not vanish, despite supersymmetry, because a finite force cannot move an infinitely massive object. Naively, one might have expected the force for $\nu = 1/2$ to be $(1/2)NT_f$ because in this case the D5-brane is connected to the D3-branes by N half-strings (at least if $Z_\infty \neq 0$). In fact, the force vanishes when $\nu = 1/2$, as it must in order to avoid contradiction with previous results for the T-dual D0-D8 system. Given this, it is understandable that the non-zero force for $\nu = 1$ is $(1/2)NT_f$ rather than NT_f (as one might naively have expected). The fact that the force remains at this value for all $\nu > 1$ is a reflection of the fact that the addition of more strings has no effect on the D3-D5 dynamics; they pass straight through the D3-brane, as required by the s-rule.

There are various dual manifestations of the HW effect. One is the creation of M2-branes when two 'linked' M5-branes cross [1, 21]. As in the D5-D3 case, the M5-worldvolume geometry is determined in terms of a function $Z(\rho)$ that gives the distance on the axis separating the M5-probe from the background in terms of radial distance on the probe. Given that the M5-branes become D4-branes when reduced on their common direction, the same results should be obtained by considering a IIA D4-brane in a D4 background (although justification of the supergravity approximation entails a return to the M-theory description). This is indeed the case; the equation for $Z(\rho)$ was found by energy minimization of a D4 probe in [8] and from supersymmetry preservation of an M5-brane in [7]. The solution of this differential equation is given implicitly by the algebraic relation[4]

$$Z = Z_\infty - \frac{L^3}{\rho^2}\left[\frac{Z}{\sqrt{\rho^2+Z^2}} + 1 - 2\nu\right]. \tag{6.1}$$

Given the freedom of sign for Z_∞ one may again take $\nu \geq 1/2$ without loss of generality. Analysis of the function $Z(\rho)$ for various choices of the constants Z_∞ and ν yields results that are qualitatively similar to those of the D5-D3 case. The analogue of the function (3.3) turns out to be [8]

$$D_\nu(\theta) = NT_2\left[\nu - \frac{1}{2}\left(1 - \cos\theta - \cos\theta\sin^2\theta\right)\right], \tag{6.2}$$

where T_2 is the M2-brane tension and (as in the D5-D3 case) $\tan\theta = \rho/Z$. Although this is a quite different function of θ from the one of (3.3), it has the same property,

$$D_\nu(\pi) = D_{\nu-1}(0)\,, \tag{6.3}$$

that is crucial to the worldvolume realization of the s-rule.

For $\nu = 1/2$ the M5-brane interpolates between a Minkowski vacuum embedded in the M-theory vacuum and an $AdS_3 \times S^3$ vacuum embedded in $AdS_7 \times S^4$, so this is another example of vacuum interpolation 'on the brane'. One would again expect an enhanced supersymmetry associated to some AdS_3 supergroup with 16 supersymmetries. The simplest candidate supergroup is $\mathrm{OSp}(4|2)\times\mathrm{OSp}(4|2)$, but there are other possibilities; we leave to the future the verification of enhanced supersymmetry in this case and the determination of the precise invariance supergroup.

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[4]This corrects a sign in [7, eq. (3.58)], which was stated there as having been transcribed from [8]. The corrected formula is indeed equivalent to the one given in [8] when $Z < 0$, but not when $Z > 0$. We believe that the formula given here is the correct one for all Z because it has the expected property that $Z \to -Z$ yields the same formula but with $Z_\infty \to -Z_\infty$ and $\nu \to 1-\nu$.

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